VSWR

Consider again the **voltage** along a terminated transmission line, as a function of **position** z:

$$V(z) = V_0^+ \left[e^{-j\beta z} + \Gamma_L e^{+j\beta z} \right]$$

Recall this is a **complex** function, the magnitude of which expresses the **magnitude** of the **sinusoidal signal** at position z, while the phase of the complex value represents the relative **phase** of the sinusoidal signal.

Let's look at the **magnitude** only:

$$\begin{aligned} |V(z)| &= |V_0^+| \, |e^{-j\beta z} + \Gamma_L e^{+j\beta z}| \\ &= |V_0^+| |e^{-j\beta z}| |1 + \Gamma_L e^{+j2\beta z}| \\ &= |V_0^+| |1 + \Gamma_L e^{+j2\beta z}| \end{aligned}$$

ICBST the **largest** value of $|V(z)|$ occurs at the location z where:

$$\Gamma_L e^{+j2\beta z} = |\Gamma_L| + j0$$

while the **smallest** value of $|V(z)|$ occurs at the location z where:

$$\Gamma_L e^{+j2\beta z} = -|\Gamma_L| + j0$$

As a result we can conclude that:

$$\left|V(z)\right|_{max} = \left|V_0^+\right|\left(1 + \left|\Gamma_L\right|\right)$$

$$\left|V(z)\right|_{min} = \left|V_0^+\right|\left(1 - \left|\Gamma_L\right|\right)$$

The ratio of $\left|V(z)\right|_{max}$ to $\left|V(z)\right|_{min}$ is known as the **Voltage Standing Wave Ratio** (VSWR):

$$\text{VSWR} \doteq \frac{\left|V(z)\right|_{max}}{\left|V(z)\right|_{min}} = \frac{1 + \left|\Gamma_L\right|}{1 - \left|\Gamma_L\right|} \qquad \therefore \qquad 1 \leq VSWR \leq \infty$$

Note if $\left|\Gamma_L\right| = 0$ (i.e., $Z_L = Z_0$), then VSWR = 1. We find for this case:

$$\left|V(z)\right|_{\max} = \left|V(z)\right|_{\min} = \left|V_0^+\right|$$

In other words, the voltage magnitude is a **constant** with respect to position *z*.

Conversely, if $\left|\Gamma_L\right| = 1$ (i.e., $Z_L = jX$), then VSWR = ∞. We find for **this** case:

$$\left|V(z)\right|_{\min} = 0 \qquad \text{and} \qquad \left|V(z)\right|_{\max} = 2\left|V_0^+\right|$$

In other words, the voltage magnitude varies **greatly** with respect to position *z*.

As with **return loss**, VSWR is dependent on the **magnitude** of Γ_L (i.e, $|\Gamma_L|$) **only** !